# Staploe Education Trust <br> Excellence through partnership 



## Calculations Policy

| Policy owned by Kennett and The Shade Advisory Body |  |
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| Chair Signature: | J Humphrey |



## Progression towards a standard written method of calculation

## INTRODUCTION

This calculation policy has been written in line with the programmes of study taken from the revised National Curriculum for Mathematics (2014). It provides guidance on appropriate calculation methods and progression. The content is set out in progressive steps under the following headings: addition, subtraction, multiplication and division.
Statements taken directly from the programmes of study are listed in bold at the beginning of each section.

Children will use mental methods as their first port of call when appropriate, but for calculations that they cannot do in their heads, they will need to use an affective written method confidently and accurately.

## AIMS OF THE POLICY

- To ensure consistency and progression in our approach to calculation
- To ensure that children develop an efficient, reliable, formal written method of calculation for all operations
- To ensure that children can use these methods accurately with confidence and understanding


## HOW TO USE THIS POLICY

- Use the policy as the basis of planning but ensure you use previous or following steps guidance to allow for personalised learning
- Always use Assessment for Learning to identify suitable next steps in calculation for groups of children to the previous stage in calculation
- Always use suitable resources, models and images to support children's understanding of calculation and place value, as appropriate
- Encourage children to make sensible choices about the methods they use when solving problems.


## Addition

## Addition - Stage 1: Build it, Draw it, Say it, Write it

Children will engage in a wide variety of songs and rhymes, games and activities.

In practical activities and through discussion they will begin to use the vocabulary involved in addition.

'You have three apples and I have four apples. How many apples altogether?'

Build it - real life objects and apparatus to explore the different models of addition.
Children begin to relate addition to combining two groups of objects, first by counting all and then by counting on from the largest number.

1. Augmentation in which two groups are combined: There are 3 footballs in the red basket 2 footballs in the blue basket. How many footballs are there altogether?
2. Aggregation in which one group is added to: Peter has 3 marbles. Harry gives Peter 1 more marble. How many marbles does Peter have now?


Draw it / Write it - Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.


Singapore Bar Model


Say it - Children will continue to practise counting on from any number e.g. 'Put five in your head and count on four.' "I have three apples, If I add four more I will have 7 apples altogether".

## Addition - Stage 2: Number lines

- Given a number, identify one more
- Read, write and interpret mathematical statements involving addition (+) and the equals (=) sign
- Show that addition of two numbers can be done in any order (commutative)
- Add one-digit and two-digit numbers within 20, including zero
- Solve missing number problems e.g. $10+\square=16$

Initially use a number line to count on for addition, counting on from the largest number:


$$
8+5=13
$$

'Put your finger on number eight. Count on (count forwards) five.'

Continue to practise counting on from the largest number for addition with totals within 20.
When confident, children should recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

Addition - Stage 3: Empty number lines
Ensure children are confident with using a marked number line before moving on to an empty number line.

- Add numbers using concrete objects, pictorial representations, and mentally, including:
- a two digit number and ones
- a two digit number and tens
- two two-digit numbers
- three one-digit numbers

Steps in addition can be recorded on an empty number line. The steps often bridge through a multiple of 10 .

Use in conjunction with a 100 square to show jumps of tens.
$34+23=57$
'Put the biggest number first (34), and then partition the smaller number (23 = $20+3$ ) and count on: $34+20+3$.'

$8+7=15$


## Addition - Stage 4 (Optional)

- Estimate the answer to a calculation and use inverse operations to check answers
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Add numbers with up to three digits, using formal written method of columnar addition

Children should be able to partition 2-digit/3-digit numbers before adding them together. Some children find it useful to record partitioning this way using concrete objects for support.

'Partition the numbers into tens and ones/units.
Add the tens together and then add the ones/units together.
Recombine to give the answer'.

Then move on to calculations that bridge the tens:
$43+29=40+3+20+9$
$40+20=60$
$3+9=12$
$60+12=72$
$43+29=72$
This is an alternative way of recording the partitioning method.

- Estimate the answer to a calculation and use inverse operations to check answers
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Add numbers with up to three digits, using formal written method of columnar addition


## Addition-Stage 5-Partitioning

- Estimate and use inverse operations to check answers to a calculation
- Solve addition two-step problems in contexts, deciding which operations and methods to use and why.

Initially use calculations where it has not been necessary to bridge across the tens or hundreds:

'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together. Recombine to give the answer.'

Children should be able to partition 2-digit/3-digit numbers before adding them together.

Some children find it useful to record partitioning in this way:


This will lead into the formal written method...

## Addition - Stage 6 (optional)

- Add whole numbers with more than 4 digits, including using formal written methods (columnar addition)
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Solve addition multi-step problems in contexts, deciding which operations and methods to use and why.

Use the language of place value to ensure understanding. Add the least significant digits (ones/units) together first and then the tens then combine the two numbers.


Further develop the formal written method of addition, with three-digit and four-digit numbers.


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Addition - Stage 7-Column method
In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.

TU
67
$\begin{array}{r}+\quad 24 \\ \hline\end{array}$
Model with Base 10 apparatus first:


Exchange ten 1s for a 10.

Use the language of place value to ensure understanding: ' 3 add 3 equals 6 '. Write 6 in the ones/units column. ' 60 add 60 equals 120 ' (no ones/units, 2 tens and 1 hundred), add the 2 to the tens column and 'carry' one (100) across into the hundreds column above. 100 add 100 and the hundred that we 'carried' equals 326 .


This can be extended to the addition of thousands and decimals.

Our aim is that by the end of Y6, children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.

## Models and Images

It is important for children to have experienced a range of models and images to ensure that they develop fluency and a secure understanding of addition and place value.


## Subtraction



Subtraction - Stage 1
Children will engage in a variety of counting songs and rhymes and practical activities.
In practical activities and through discussion they will begin to use the vocabulary associated with subtraction.

They will find one less than a given number.
They will begin to relate subtraction to 'taking away' using objects to count 'how many are left' after some have been taken away.

'Take three apples away. How many are left?'
Subtraction - Stage 2

- Given a number, identify one less
- Read, write and interpret mathematical statements involving subtraction
$(-)$ and the equals ( $=$ ) sign
- Subtract one-digit numbers

Introduce complementary addition to find differences (only use for small differences). The use of models is extremely important here to understand the idea of "difference".

Counting on to find a small difference:
Count up from the smallest number to the largest to find the difference using resources, e.g. cubes, beads, number tracks/lines:

The difference between seven and five is two.

$7-5=2$

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures and objects, etc.

Comparing two sets (comparison or difference):


Singapore Bar Model:

## - Peter has 5 pencils and 3 erasers. How many more pencils than erasers does he have?



Subtraction - Stage 3: The number line

- Subtract numbers using concrete objects, pictorial representations, and mentally. Children will begin to count back from a given number.
- Subtract one- digit and two-digit numbers within 20, including zero
- Solve missing number problems e.g. $20-\square=15$

Children will continue to practise counting back from a given number.
Initially use a number line to count back for subtraction:


$$
12-3=9
$$

'Put your finger on number twelve. Count back three.'

## Subtraction - Stage 4

- Estimate the answer to a calculation and use inverse operations to check answers
- Solve problems, including missing number problems, using number facts, place value, and more complex subtraction.
- Subtract numbers with up to three digits, using formal written method of columnar subtraction

Counting on to find a small difference.
Introduce complementary addition to find differences (only use for small differences). The use of models is extremely important here to understand the idea of "difference"

Count up from the smallest number to the largest to find the difference.

'The difference between 9 and 12 is 3 '

## Subtraction - Stage 5

- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Ensure that children are confident with the methods outlined in the previous year's guidance before moving on. Use in conjunction with a 100 square to show jumps of tens.

## Backwards bounce



$$
87-23=64
$$

'Partition the smaller number ( $23=20+3$ ) and count back.'

## Subtraction - Stage 6

## Counting on to find a small difference - The triple jump!

Once the children are confident to count up from the smallest number to the largest to find the difference using partitioning, further develop this method into the triple jump;


## Subtraction - Stage 7- expanded layout (no exchanging)

Expanded layout leading to column method. Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens are placed under tens.


## Subtraction - Stage 8

- Subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)

Introduce the formal written method with the calculation - column subtraction. Use two-digit numbers when introducing this method, initially:


Use the language of place value to ensure understanding:
'Eight subtract six, seventy subtract forty.'

## Subtraction - Stage 9

Ensure that children are confident with the methods outlined in the previous stage's guidance before moving on.
Use the language of place value to ensure understanding and use base-ten materials, if necessary. In this example it has been necessary to exchange from the hundreds column.

## Step 1:

$$
\begin{array}{l|l}
500+60+3 \\
200+70+1 \\
\hline
\end{array}
$$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixtythree'. Then discuss the hundreds, tens and ones components of the number, and how 500 +60 can be partitioned into $400+160$. The subtraction of the tens becomes ' 160 minus 70 '.

## Step 2



Model exchanging (adjusting) with Base 10 first:

$\qquad$


This can be extended to the subtraction of thousands and decimals.
Our aim is that by the end of Y6, children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.

## Multiplication

Before reaching the calculation stage of multiplication, children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving doubling.

Children will then move on to count repeated groups of the same size in practical contexts. They will use the vocabulary associated with multiplication in practical contexts. They will solve practical problems that involve combining groups of 2,5 or 10. They will continue to solve problems in practical contexts and develop the language of early multiplication, with appropriate resources.

## Multiplication - Stage 1

Children will use a range of vocabulary to describe multiplication and use practical resources, pictures, diagrams and the ' $x$ ' sign to record.

Combining Groups - repeated addition.

'How many crayons altogether?'

$$
‘ 5+5+5=15 ’
$$

'3 groups of 5' '3 times 5'

$$
\text { ' } 3 \times 5=15 \prime \quad \quad ' 5 \times 3=15 \prime
$$

Solve one-step problems involving multiplication, by calculating the answer using concrete objects and pictorial representations with the support of the teacher. Children use real life objects and apparatus to explore the different models of multiplication.


Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.


Singapore Bar Model:
Emily has 7 stickers, Joe has six times as many stickers


7

Repeated addition using a number line
Repeated addition can be shown easily on a number line:


3 times 5 is $5+5+5=15$ or 3 lots of 5 or $5 \times 3$


Children should know that $3 \times 5$ has the same answer as $5 \times 3$ (the commutative law of multiplication). This can also be shown on the number line.


## 0000008 00000000



## Multiplication - Stage 2: Arrays

- Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication $(X)$ and equals (=) signs
- Show that multiplication of two numbers can be done in any order (commutative)
- Solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication, including problems in contexts.


$$
\begin{gathered}
5+5+5=15 ' \\
\text { '3 rows of } 5 \text { ' } \\
\text { '3 groups of } 5 \text { ' } \\
\text { '5 groups of } 3 \text { ' } \\
\text { ' } 5 \times 3=15^{\prime} \\
' 3 \times 5=15^{\prime}
\end{gathered}
$$

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method. Show children real-life arrays first:


Multiplication - Stage 3: The Grid Method.

- Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- Solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which $\mathbf{n}$ objects are connected to $\mathbf{m}$ objects.

Before using the grid method, children should have a secure understanding of place value and be able to multiply confidently by multiples of 10 . This should not be taught through how many zeros are placed on the end of a number; children should understand that as the digit moves to the left on a place value grid, its value becomes 10 times greater and the zero is a place holder.


Partitioning method for multiplication of a 2-digit number by a 1-digit number - grid method.
Continue to use arrays, leading into the grid method for multiplication.
$14 \times 6=$

$(6 \times 10)+(6 \times 4)$
$60+24$
84

Then-

'Partition 43 into 40 and 3 , then multiply each number by 6 . Add the numbers together'
Multiplication - Stage 4

- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling
problems and harder correspondence problems such as $\mathbf{n}$ objects are connected to m objects.

When confident, children will further develop the grid method for multiplying two 2-digit numbers.


Partition 43 into $40+3$, and partition 65 into $60+5$.
If children are confident, continue to develop grid method multiplication with threedigit numbers multiplied by a one-digit number.

Multiplication - Stage 5

- Multiply whole numbers and those involving decimals by 10, 100 and 1000
- Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- Solve problems involving multiplication, including scaling by simple fractions and problems involving simple rates.
- Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- Pupils practise and extend their use of the formal written methods of short multiplication

Expanded/long multiplication:


Use the language of place value to ensure understanding. Add the partial products.

## Multiplication - Stage 6

Short multiplication -contracted form

' 3 times 6 is 18 , we write the 8 in the 1 s column and the 1 ten in the 10 s column above the line like we do for addition. Now we work out $3 \times 20,2$ tens, that's 6 tens, plus the 1 ten we had from multiplying the 1 s , so that's 7 tens, so we write 7 in the 10 s column.
Then we work out $3 \times 300$, that's nine 100 s and we write this in the 100 s column'

'Stage 1- multiply10 $\times 329$. This is easy as we just find 10 lots of 329 and write it down. That was nice because the digits just move along to the left. Stage 2-3 $\times 329$. We use short multiplication to work out $3 \times 329$, starting with $3 \times 9$ (the 1 s) but instead of the two 10 s being written above the line like we have done before, we just write them in front of the 7 to show 27 . We must then remember to add these two 10 s when we work out the next part: $3 \times 20$ (the 10 s ). This is because if we leave another line and write the 2 there like we do for short multiplication, it will be confusing when we do the addition at the end. Repeat Stage 2 for the 100s and explain that if children find this difficult to begin with, they can just use short multiplication to work out $3 \times 329$ at the side and write the answer under 3290 . Stage 3, addition. We will add these two numbers. We need to leave a blank row for this and add as normal using column

## Division

## Division - Stage 1-Concrete

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving halving and sharing. Children use real life objects and apparatus to explore the different models of division.

Share the apples between two people.

'Half of the apples for you and half of the apples for me.'


## Division - Stage 2

- Solve problems involving division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Children will move on from sharing to grouping in a practical way.

'Put 12 apples into groups of 2 - how many groups are there?'



Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.
$12 \div 4=3$


Use arrays to support early division.
Children will further develop division by using a range of vocabulary to describe division and use practical resources, pictures, diagrams and the $\div$ sign to record, using multiples that they know.

## Division - Stage 3

- Calculate mathematical statements for division within the multiplication tables and write them using the division ( $\div$ ) and equals ( $=$ ) signs
- Show that division of one number by another cannot be done in any order

Once children have developed the grouping strategy, they are introduced to the grouping grid to record. Remainders can be taught as 'how many left over'?

'How many times can I fit (groups of) 4 into 27?

This can be supported by using jumps on an empty number line;


Use the Singapore Bar as a representation to solve division problems. This model can be used at ALL STAGES as a tool for children to work out what calculation they need to carry out:

Joe has 6 times as many stickers as Emily. Joe has 48 stickers. How

many stickers does Emily have?

We know that Joe has 48 stickers in total.

The model shows us that we must calculate $48 \div 6$


Joe has 6 times as many stickers as Emily. Joe has 48 stickers. How many stickers does Emily have?

It is clear then that Emily also has 8 stickers...

48

| Joe's 8 <br> Stickers | Joe's 8 <br> Stickers | Joe's 8 <br> Stickers | Joe's 8 <br> Stickers | Joe's 8 <br> Stickers | Joe's 8 <br> Stickers |
| :--- | :--- | :--- | :--- | :--- | :--- |

Emily's
Stickers

$$
48 \div 6=8
$$

The array is an
image for division
too


This representation can be used to show why division calculations are sometimes laid out like this. We are looking for the missing side of the array.

## Division - Stage 4

- Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- Solve problems, including missing number problems, involving division, including positive integer scaling problems and correspondence problems in which $\mathbf{n}$ objects are connected to $\mathbf{m}$ objects.

Introduce a formal layout using multiplication/division facts that the children know:
$24 \div 3=8$ this can also be recorded as...

'Twenty four divided by three equals eight.'
'How many threes are there in twenty four?'

Division using partitioning - Find the hunk! (two digits divided by one digit)


The
Hunk! Chunk


## Division - Stage 5

This can be further developed into; Mega hunk! (Three digits divided by one digit)

## $394 \div 6=65 \mathrm{r} 4$

Mega
Hunk!

## Chunk

## $360+34$



Partitioning can be practised in a variety of ways.

## $98 \div 7=14$

$98=70+28 \quad$ Partition 98 into 70 and 28
$70 \div 7=10$
$28 \div 7=4$
$10+4=14$

$$
7 \longdiv { 1 0 + 4 = 1 4 }
$$

This can be supported by chunking on a number line. Relate to the inverse of multiplication. Ask, how many 5 s in 70 ? 10 lots of 5,4 lots of 5 . How many lots of 5 altogether? 14.


Division with remainders:
$72 \div 5=14 r 2$


Division - Stage 6 - Children confident at stage 5 can move on to formal methods

- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Divide whole numbers and those involving decimals by 10, 100 and 1000
- Solve problems involving division including using their knowledge of factors and multiples, squares and cubes
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- Solve problems involving division, including scaling by simple fractions and problems involving simple rates.
- Pupils practise and extend their use of the formal written methods of short division

The formal written method of short division:


Remainders can also be expressed as a fraction.

## Division - Stage 7

- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context


## Long division



Multiples of the divisor (15) have been subtracted from the dividend (432) ' 20 (lots of $15)+8($ lots of 15$)=2812$ is the remainder'


Our aim is that by the end of Y6 children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.

